MAT 1341 Assignment 1

Summer 2007

Due date: May 23 6:00pm

Instructor: Charles Starling

Family Name:_____

First Name:_____

Student Number: _____

Question	Response	Points
1		
2		
3		
4		
5		
6		
7	–	
8	–	
Total	–	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. Read each question carefully, and answer all questions in the space provided after each question. For questions 7 and 8, you may use the back of the pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 6 are worth 1 point each, and no part marks will be given. However, you must show some work to obtain the point. Simply writing the correct answer will earn you no points.
3. Questions 7 and 8 are worth 4 points, and part marks can be earned. **The correct answers here require justification written legibly and logically; you must convince me that you know why your solution is correct**.
4. Submit this assignment to me on May 23rd in class. Assignments will be accepted at the beginning of class with no penalty. Until the end of class, papers will be accepted with a 1 mark penalty. After the end of the class assignments will not be accepted and their weight will be transferred to the final exam.

1. Which two the following are NOT subspaces of $\mathbb{R}^4$?

$S = \{(x, y, z, w) \mid xyz \geq 0\}$

$T = \{(x, y, z, w) \mid x = 0, y = 4x + 5z \text{ and } w = -z\}$

$U = \{(x, y, z, w) \mid 3x + 2y = z \text{ and } z = w\}$

$V = \{(x, y, z, w) \mid x^3 + y^2 + z = 2\}$

2. Which two of the following are subspaces of $\mathbb{F}[0, 1] = \{f | f : [0, 1] \longrightarrow \mathbb{R}\}$?

$S = \{f \in \mathbb{F}[0, 1] \mid f(x) \geq 0 \text{ for all } x \in \mathbb{R}\}$

$T = \{f \in \mathbb{F}[0, 1] \mid f(0) = 3f(1)\}$

$U = \{f \in \mathbb{F}[0, 1] \mid f(x + \frac{1}{2}) = f(x)\}$

$V = \{f \in \mathbb{F}[0, 1] \mid f(0) = f(1)^2\}$

3. True or False: if $u \times v = v \times u$ where $u, v \neq 0$, then $u = \lambda v$ for some $\lambda \in \mathbb{R}$.

4. Which of the following is a spanning set for $T = \{(x, y, z) \in \mathbb{R}^3 \mid 4x + 4y - z = 0\}$?

A. {(4,4,-1)}

B. {(4,4,0), (0,0,-1)}

C. {(1,0,4), (0,1,4)}

D. {(1,0,0), (0,1,0)}

E. {(4,0,1), (0,1,0)}

5. True or False: if u and v are vectors in $\mathbb{R}^3$, then $\text{span}\{u, v, u \times v\}$ is all of $\mathbb{R}^3$.

6. Which two of the following functions in $\mathbb{F}(\mathbb{R}) = \{f | f : \mathbb{R} \longrightarrow \mathbb{R}\}$ is in $\text{span}\{\cos^2 x, \sin^2 x, x, x^2\}$?

A. $3 + x$

B. $x \sin x$

C. $x^3 + \cos x$

D. $x^2 + \sin^2 x$

E. $x \sin^2 x + x^2$

7. Let $u_0 = (4, 1, -1)$, and let

$$W = \{v \in \mathbb{R}^3 \mid v \times u_0 = 0\}$$

a) Show that W is a subspace of $\mathbb{R}^3$.

b) Carefully show that $v \times u_0 = 0 \iff v$ is a scalar multiple of u_0.

c) Find a spanning set for W.

d) Give a complete geometric description of W.

8. Let

$$M_1 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \; ; \; M_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \; ; \; M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \; ; \; M_4 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

a) Show that $\text{span}\{M_1, M_2, M_3, M_4\}$ is NOT all of $\mathbf{M}_{22} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$.

(**Hint:** assume that the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is in the span and come to a contradiction.

b) Show that $\{M_1, M_2, M_3, M_4\}$ is a linearly dependent set.