MAT 1341 Assignment 1

Winter 2008

Due date: Feb 7 7:00pm

Instructor: Charles Starling

Family Name:______________________

First Name:______________________

Student Number: ____________________

Question	Response	Points
1	–	
2	–	
3	–	
4		
5		
6		
Total	–	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. **Print this assignment single sided; write all your answers on the printout**. For questions 1 to 3, you may use the back of the pages if necessary, but be sure to indicate to the marker that you have done this.
2. Question 1 is worth 25 points, question 2 is worth 20 points and question 3 is worth 25 points. **The correct answers here require justification written legibly and logically; you must convince me that you know why your solution is correct.**
3. Questions 4, 5, and 6 are multiple choice and are worth 10 points each. No justification for your answer is needed here - **write your answers on this page in the box given**.
4. Submit this assignment to me on Feb 7 in class. Assignments will be accepted at the beginning of class with no penalty. Until the end of class, papers will be accepted with a 1 mark penalty. **After the end of the class assignments will not be accepted**. If you cannot make it to the class then you may drop them off at my office anytime before class on Feb 7.

1. For each of the following statements, indicate whether they are true or false. If a statement is false, give a counterexample. If it is true, give a short argument.

(i) Suppose we have a linear system consisting of m equations in n variables. Then if $m \geq n$, the system can have at most one solution.

(ii) Let A, B be matrices such that the product AB is defined. Then $\text{rank}(AB) = \text{rank } A$.

(iii) If A, B are $n \times n$ matrices and $AB = 0$ then at least one of A, B must have determinant zero.

(iv) If an $n \times n$ matrix A satisfies $AB = BA$ for every $n \times n$ matrix B, then A must be the identity matrix.

(v) If the system $Ax = b$ has no solutions, then the system $Ax = 0$ has only the trivial solution.

2. Find all solutions to the following system of equations:

$$
\begin{aligned}
2x_1 + 3x_2 + x_3 + 4x_4 - 9x_5 &= 17 \\
x_1 + x_2 + x_3 + x_4 - 3x_5 &= 6 \\
-x_1 - 2x_2 - 2x_3 + 4x_4 &= -9 \\
2x_1 + 2x_2 + 2x_3 + 3x_4 - 8x_5 &= 14
\end{aligned}
$$

3. Find all values of b for which the following homogeneous system has a nontrivial (i.e. non-zero) solution:

$$\begin{aligned} bx_1 - bx_3 &= 0 \\ x_1 + (b+1)x_2 + 2x_3 &= 0 \\ bx_1 + (2b+2)x_2 &= 0 \end{aligned}$$

4. Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{bmatrix}$$

A. A is not invertible.

B. A is invertible, and the largest entry of A^{-1} is 119.

C. A is invertible, and the largest entry of A^{-1} is 69.

D. A is invertible, and the largest entry of A^{-1} is 27.

E. None of the above.

5. An $n \times n$ matrix D is called **diagonal** if all nonzero entries are on the main diagonal. Precisely, if $D = [d_{ij}]$ then $d_{ij} \neq 0$ implies $i = j$. For example, the identity matrix is diagonal. Consider the following statements.

1. Any diagonal matrix is invertible.
2. If A and B are diagonal and of equal size then AB is also diagonal.
3. Any diagonal matrix is equal to its own adjoint.
4. Any element on the main diagonal of a diagonal matrix is an eigenvalue of that matrix.

A. All statements are true.

B. Statements 1, 4 are true, but 2, 3 are false.

C. Statements 1, 4 are false, but 2, 3 are true.

D. Statements 2, 4 are true, but 1, 3 are false.

E. Statements 2, 4 are false, but 1, 3 are true.

F. All statements are false.

G. None of the above.

6. Consider the following matrix A:

$$A = \begin{bmatrix} 5 & 6 & -7 \\ 0 & 1 & 0 \\ 6 & 6 & -8 \end{bmatrix}$$

Which of the following is true?

A. A has three distinct eigenvalues.

B. A has two distinct eigenvalues.

C. A has exactly one eigenvalue.

D. A has no eigenvalues.