MAT 1341 Assignment 2

Summer 2007

Due date: June 27th 6:00pm

Instructor: Charles Starling

Family Name:_______________________

First Name:_________________________

Student Number: ____________________

Question	Response	Points
1		
2		
3		
4		
5	–	
6	–	
7	–	
Total	–	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. Read each question carefully, and answer all questions in the space provided after each question. For questions 5 to 7, you may use the back of the pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 4 are worth 2 points each, and you must show some work to obtain the points. Simply writing the correct answer will earn you no points.
3. Question 5 is worth 4 points, question 6 is worth 3 points, and question 7 is worth 8 points. **The correct answers here require justification written legibly and logically; you must convince me that you know why your solution is correct.**
4. Submit this assignment to me on June 27th in class. Assignments will be accepted at the beginning of class with no penalty. Until the end of class, papers will be accepted with a 1 mark penalty. After the end of the class assignments will not be accepted and their weight will be transferred to the final exam.

1. Let u, v, w, and z be vectors in a vector space V. Which of the following are true?

(1) If $\{u, v\}$ and $\{w, z\}$ are both linearly independent, then $\{u, v, w, z\}$ is linearly independent.

(2) If $\{u, v\}$ is linearly independent, then so is $\{u, u + v\}$.

(3) If $\{u, v, w\}$ is linearly independent, then so is $\{u, w\}$.

(4) If one of $\{u, v\}$ and $\{w, z\}$ is linearly dependent, then $\{u, v, w, z\}$ is linearly dependent.

(5) If $\{u, v, w\}$ is linearly independent, then $\{u + v, u + w, v + w\}$ is linearly independent.

A. (1) and (2)

B. (3) and (4)

C. (1), (3), and (4)

D. (2), (3) and (4)

E. All except (1) are true

2. For what values of b is the following system consistent?

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ 3x_1 + x_3 &= 3 \\ 5x_1 - 2x_2 + 3x_3 &= b \end{aligned}$$

A. All real numbers

B. 0

C. 5

D. 5 and 0

E. 1

3. Which two of the following are true:

A. If $A^2 = 0$, the zero matrix, then A is not invertible.

B. If any column is deleted from a matrix in row reduced echelon form, then the resulting matrix is still in row reduced echelon form.

C. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$,then the second row of A^{-1} is $[0 \;\; 1 \;\; 1/2]$

D. If three $n \times n$ matrices A, B and C satisfy $AB - BA = C$ then $ABA = BA + CA$.

4. Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$. What is the first row of A^{10001}?

A. $[1, 0, 0, 0, 0]$

B. $[10001, 0, 0, 10001, 20002]$

C. $[1, 0, 0, 1, 2^{10001}]$

D. $[1, 0, 0, 1, 2]$

5.Find a homogeneous system whose solution set is spanned by:

$$\{u_1, u_2, u_3\} = \{(1, -2, 0, 3), (1, -1, -1, 4), (2, -1, -3, 9)\}$$

6. Find the general solution of the following linear system:

$$\begin{aligned} x_1 - 2x_2 + x_3 + 3x_4 - x_5 &= 1 \\ -3x_1 + 6x_2 - 4x_3 - 9x_4 + 3x_5 &= -1 \\ -x_1 + 2x_2 - 2x_3 - 4x_4 - 3x_5 &= 3 \\ x_1 - 2x_2 + 2x_3 + 2x_4 - 5x_5 &= 1 \end{aligned}$$

7. a) Suppose that A and B are square matrices and that AB is invertible. Prove that A and B are invertible.

b) Suppose that A is an invertible matrix and that $AC = CA$ for some matrix C. Show that $A^{-1}C = CA^{-1}$.

c) Suppose that B is a 2×2 matrix such that $AB = BA$ for *every* 2×2 matrix A. Show that $B = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ for some $a \in \mathbb{R}$.

d) A matrix M is said to be *symmetric* if $M = M^t$. Show that if A, B and AB are symmetric matrices, then $AB = BA$.