

## MAT 1341D Assignment 2

Winter 2008

Due date: March 11 7:00pm

Instructor: Charles Starling

Family Name: \_\_\_\_\_

First Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

Question	Response	Points
1		
2		
3		
4		
5	–	
6	–	
Total	–	

### PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. Read each question carefully, and answer all questions in the space provided after each question. For questions 5 and 6, you may use the back of the pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 4 are worth 2 points each. Part marks will be given for question 1, but not for 2, 3, and 4.
3. Questions 5 and 6 are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically; you must convince me that you know why your solution is correct.**
4. Submit this assignment to me on March 11th in class. Assignments will be accepted at the beginning of class with no penalty. Until the end of class, papers will be accepted with a 2 mark penalty. After the end of the class assignments will not be accepted.

1. For each of the following, indicate whether it is a subspace of  $\mathbb{R}^4$  (half a mark each).

$$S = \{(x, y, z, w) \mid xyz \geq 0\}$$

$$T = \{(x, y, z, w) \mid x = 0, y = 4x + 5z \text{ and } w = -z\}$$

$$U = \{(x, y, z, w) \mid 3x + 2y = z \text{ and } z = w\}$$

$$V = \{(x, y, z, w) \mid x^3 + y^2 + z = 2\}$$

2. What is the shortest distance between the point  $P = (1, 1, 1)$  and the plane given by the equation  $2x + 3y - z = 8$ ?

A.  $\sqrt{14}$

B.  $\frac{2}{7}$

C.  $\frac{2\sqrt{14}}{7}$

D.  $\sqrt{3}$

E. 8

3. Which of the following is a spanning set for  $T = \{(x, y, z) \in \mathbb{R}^3 \mid 5x + 3y + z = 0\}$ ?

- A.  $\{(5,3,1)\}$
- B.  $\{(5,3,0), (0,0,1)\}$
- C.  $\{(1,0,5), (0,1,3)\}$
- D.  $\{(1,0,-5), (0,1,-3)\}$
- E.  $\{(-1,0,-5), (0,-1,-3)\}$

4. Let  $v_1 = (1, 1, 1)$ ,  $v_2 = (3, -1, -1)$  and  $v_3 = (-1, 0, 4)$ . Let  $V$  be the volume of the parallelepiped whose edges are  $v_1, v_2$  and  $v_3$ . Let  $A$  be the area of the parallelogram whose edges are  $v_1$  and  $v_2$ . Then which of the following is true?

- A.  $\|v_2\| < V < A$
- B.  $\|v_2\| < A < V$
- C.  $A < \|v_2\| < V$
- D.  $A < V < \|v_2\|$
- E.  $V < A < \|v_2\|$
- F.  $V < \|v_2\| < A$

5. Let  $u_0 = (5, -1, 1)$ , and let

$$W = \{v \in \mathbb{R}^3 \mid v \times u_0 = 0\}$$

- a) Show that  $W$  is a subspace of  $\mathbb{R}^3$ .
- b) Carefully show that  $v \times u_0 = 0 \iff v$  is a scalar multiple of  $u_0$ .
- c) Find a spanning set for  $W$ .
- d) Give a complete geometric description of  $W$ .

6. Decide whether the following statements are true or false. If the statement is true, prove it. If it is false, provide a counterexample showing that it is false.

i) If  $u \times v = v \times u$  where  $u, v \neq 0$ , then  $u = \lambda v$  for some  $\lambda \in \mathbb{R}$

ii) If  $U$  and  $V$  are both subspaces of  $\mathbb{R}^n$ , then  $U \cap V$  (the intersection of  $U$  and  $V$ ) is a subspace of  $\mathbb{R}^n$ .

iii) The intersection of two planes in  $\mathbb{R}^3$  is always a line.

iv) Let  $u_1$  and  $u_2$  be vectors in  $\mathbb{R}^n$ . If  $u_1 \cdot v = u_2 \cdot v$  for some nonzero  $v \in \mathbb{R}^n$ , then  $u_1 = u_2$ .