MAT 1341D Assignment 3

Winter 2008

Due date: April 10, 7pm

Instructor: Charles Starling

Family Name:________________________

First Name:__________________________

Student Number: ____________________

Question	Response	Points
1		
2		
3		
4		
5	–	
6	–	
Total	–	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. Read each question carefully, and answer all questions in the space provided after each question. For questions 5 and 6, you may use the back of the pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 4 are worth 2 points each, and no part marks will be given. In question 1, $\frac{1}{2}$ point will be given for each correct part.
3. Questions 5 and 6 are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically; you must convince me that you know why your solution is correct.**
4. Submit this assignment to me on April 10th at the beginning of class. Assignments will be accepted at the beginning of class with no penalty. Until the end of class, papers will be accepted with a 2 mark penalty. After the end of the class assignments will not be accepted. If you cannot make it to class that day, you can drop off the assignment at my office before 7pm.

1. For each of the following, answer the following questions: Is it a basis for $\mathbb{R}^3$? Is it orthonormal? (You don't have to justify your answers.)

$S = \{\frac{1}{\sqrt{2}}(1,0,1), (0,1,0), \frac{1}{\sqrt{2}}(-1,0,1)\}$
$T = \{(0,0,1), \frac{1}{\sqrt{2}}(0,1,1), \frac{1}{\sqrt{3}}(1,1,1)\}$
$U = \{(0,0,1), (1,0,0), \frac{1}{\sqrt{2}}(1,0,-1)\}$
$V = \{(0,0,1), \frac{1}{\sqrt{3}}(1,1,-1), \frac{1}{\sqrt{2}}(-1,0,1)\}$

2. Let A be a matrix with row echelon form R. Which of the following is false?

A. A and R have the same row space.
B. A and R have the same column space.
C. A and R have the same kernel.
D. A and R have the same rank.

3. Let $u_0 = (5, -1, 1)$. Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(v) = v \times u_0$. Then T is a linear transformation. What is the standard matrix of T?

A. $\begin{bmatrix} 5 & 5 & 5 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 5 \\ -1 & -5 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -5 \\ 1 & 5 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -5 \\ 1 & 5 & 0 \end{bmatrix}$ E. $\begin{bmatrix} 1 & -1 & 0 \\ 5 & 0 & -1 \\ 5 & -1 & 1 \end{bmatrix}$

4. Let $A = \begin{bmatrix} 1 & 4 & 0 & 3 \\ 2 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. Which of the following statements is true?

A. $\dim(\text{col}A) = 0$

B. $\dim(\ker A) = 3$

C. The rows of A are linearly dependent

D. The columns of A span $\mathbb{R}^3$

E. The rows of A span $\mathbb{R}^4$

F. There are vectors b such that $Ax = b$ has no solution

5. Consider the vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 4 \end{bmatrix}$, $v_4 = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 2 \end{bmatrix}$, and also let $U = \text{span}\{v_1, v_2, v_3, v_4\}$.

a) Find a basis for U which is a subset of $\{v_1, v_2, v_3, v_4\}$. Is $\{v_1, v_2, v_3, v_4\}$ linearly dependent or linearly independent?

b) What is the dimension of U?

c) Use the Gram-Schmidt algorithm to convert the basis you found in (a) into an orthogonal basis for U.

6. Decide whether the following statements are true or false. If the statement is true, prove it. If it is false, provide a counterexample showing that it is false.

i) If u, v are two linearly independent vectors in $\mathbb{R}^3$, then $\{u, v\}$ is an orthogonal basis for $\text{span}\{u, v\}$.

ii) There is no 3×3 matrix A such that $\text{rank}A = \dim(\ker A)$.

iii) If every row of a given matrix B has a leading 1, then the columns of B are linearly independent.

iv) Let A be a square matrix. If $\ker(A) = \{0\}$ then A is invertible.