Review Quiz

1. Suppose we have a homeogeneous linear system with 2 equations and 3 unknowns. What is the minimum number of parameters the general solution could have?

2. Is the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ diagonalizable? Why or why not?

3. Proof or counterexample: if a 2×2 matrix has only 1 eigenvalue, it is not diagonalizable.

4. Let A be a square matrix. Write down a statement equivalent to the statement "A is **not** invertible" in terms of

   (a) The rank of A.

   (b) The RRE form of A.

   (c) The solutions to the homogeneous system $Ax = 0$.

5. Find all the complex solutions to $z^2 = i$ (*hint*: let $z = re^{i\theta}$, convert i to polar form, and then solve for r and θ).

6. Find $\mathrm{Re}(z)$ for all solutions found in the above question, given that $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

7. Find the inverse of $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & i \end{bmatrix}$.

8. List the effects of column and row operations on the determinant of a matrix. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 2$, what is $\begin{vmatrix} 3a+3c & 3b+3d \\ a & b \end{vmatrix}$?

9. Proof or counterexample: if $A^2 = 0$, then $A = 0$.

10. Proof or counterexample: $\det(A^{-1}) = \frac{1}{\det A}$.