

1. Find the volume of the parallelepiped whose sides are  $(1, 4, 5)$ ,  $(2, 3, 5)$  and  $(1, 0, 0)$ .
2. Write down the formula for the cross product of two vectors  $u$  and  $v$ .

3. What is the image of  $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 8 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ ?

4. What is the kernel of  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 2 & 5 & 5 \end{bmatrix}$ ?

5. Give a complete geometric description of the set found in the above question.
6. What is the shortest distance between the point  $(1, 0, -1)$  and the plane  $x - 2y + z = 4$ ?
7. Show that the vectors  $(1, 2, -1, 1)$ ,  $(2, 4, 3, 1)$  and  $(1, 2, 4, 0)$  are linearly dependent.
8. Let  $n = (1, -2, 1)$ . By using the subspace test, show that

$$W = \{v \in \mathbb{R}^3 \mid v \cdot n = 0\}$$

is a subspace of  $\mathbb{R}^3$ .

9. Proof or counterexample: if  $\{u, v\}$  is a linearly independent set, then  $\{u, u + v\}$  is also a linearly independent set.
10. Proof or counterexample: if  $\{u, v, w\}$  is a linearly dependent set, then  $\{u, v\}$  is also a linearly dependent set.