

MAT 1341D Test 2

v1

Winter 08

March 18th

Instructor: Charles Starling

Family Name: _____

First Name: _____

Student Number: _____

Question	Points
1	
2	
3	
4	
5	
6	
7	
Total	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. You have 90 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. **Do not use your own scrap paper! Use the last page or the backs of pages for rough work.**
3. The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.
4. Questions 1 through 5 are worth 1 point each. The correct answer will get you the full point. However, if you do not have the correct answer part marks may be awarded.
5. Questions 6 and 7 require a complete solution. Question 6 is worth 9 points and question 7 is worth 6 points. Spend your time accordingly. **The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Where it is possible to check your work, do so.
7. Good luck! Bonne chance!

1. For what values of $c \in \mathbb{R}$ are the vectors $(c, 3, -4)$ and $(c, -c, 1)$ orthogonal?

2. Calculate the area of the triangle with vertices $(0, 0, 0)$, $(2, 3, 6)$ and $(1, 0, 1)$.

3. If u and v are vectors in \mathbb{R}^3 , state the formula for $\text{proj}_v(u)$, the projection of u onto v .

4. Find the shortest distance between the point $P = (1, 1, 1)$ and the plane $x + y + z = 2$.

5. Find the normal vector for the plane given by $W = \text{span}\{(1, 1, 1), (0, 1, 0)\}$.

6. Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

- a) Show that $\text{span}\{v_1, v_2, v_3, v_4\}$ is **not** all of \mathbb{R}^4 (*hint*: suppose that $[1, 0, 0, 0]^t$ is in the span and come to a contradiction.)
- b) Show that $\{v_1, v_2, v_3, v_4\}$ is linearly dependent.
- c) Let A be the matrix whose columns are v_1, v_2, v_3, v_4 , ie, $A = [v_1 \ v_2 \ v_3 \ v_4]$. Find the kernel (or nullspace) of A .
- d) Show that $\{v_1, v_2, v_3\}$ is linearly independent.

7. Decide whether the following statements are true or false. If the statement is true, prove it. If it is false, provide a counterexample showing that it is false.

i) The image (or columnspace) of a 3×3 matrix is always \mathbb{R}^3

ii) Suppose that u is a vector in \mathbb{R}^n such that $u \cdot v = 0$ for *all* vectors v in \mathbb{R}^n . Then u must be the zero vector.

iii) If $\{u, v\}$ is a linearly independent set of vectors, then $\{u, u + v\}$ must also be linearly independent.

iv) If $U = \text{span}\{u, v, w\}$, then $\{u, v, w\}$ must be linearly independent.

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